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for the metalworking trades

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Preface

The Mechnical and Metal Trades Handbook is well-suited for shop reference, tooling, machine building, maintenance and as a general book of knowledge. It is also useful for educational purposes, especially in practical work or curricula and continuing education programs.

Target Groups

- Industrial and trade mechanics
- Technical product designers
- Apprentices in above trade areas
- Practitioners in trades and industry
- Mechanical engineering students

Notes for the user

The contents of this book include tables and formulas in seven chapters as well as a table of contents, a subject index, a standard index and an international material comparison chart.

For a better overview, each of the seven chapters is preceded by an additional one-page **table of contents**.

The **tables** contain the most important guidelines, designs, types, dimensions and standard values for their respective subject areas.

Units are not specified in the legends for the **formulas** if several units are possible. However, the calculation examples for each formula use those units normally applied in practice.

Changes in the 4th edition

The standards in this edition are **current as of January 2017.** Due to new standards and technical developments, the following contents have been updated, expanded or newly added:

- Quality management and environmental management according to the latest standard. Elimination of general terms from quality management.
- Introduction to "Geometrical product specification (GPS)" for technical communication principles.
- Additional tools and partially updated standard values for machining operations.
- Additions in cost accounting.
- Representation of structuring principles and reference designation in schematic circuit diagrams according to ISO 1219 and DIN EN 81346.

Authors and publishers continue to be grateful to all users of the table handbook for notes and suggestions for improvement addressed to lektorat@europa-lehrmittel.de.

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Standards and other regulations

Standardization and standards terms

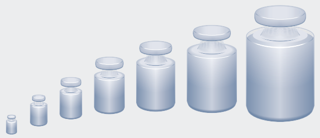
Standardization is the systematic achievement of uniformity of material and non-material objects, such as components, calculation methods, process flows and services for the benefit of the general public.

Standards term	Example	Explanation
Standard	DIN 509	A standard is the published result of the standardization work. Example: DIN 509 with shapes and dimensions of undercuts for turned parts and bores.
Part	DIN 30910-2	Standards can comprise several parts associated with each other. The part numbers are appended to the main standard number with hyphens. DIN 30910-2 describes sintered materials for filters for example, whereas Part 3 and 4 deal with sintered materials for bearings and formed parts.
Supplement	DIN 743 Suppl. 1	A supplement contains information for a standard, however no additional specifications. The supplement DIN 743 Suppl. 1, for example, contains application examples of load capacity calculations for shafts and axles described in DIN 743.
Draft	E DIN EN 10027-2 (2013-09)	Draft standards are made available to the public for examination and commenting. The new version of DIN EN 10027-2 (2015-07) with material codes for steels was available to the public as a draft for objections, for example, from September 2013 to February 2014.
Preliminary standard	DIN V 45696-1 (2006-02)	A preliminary standard contains the results of standardization, which have not been released as a standard because of certain provisos. For example, DIN V 45696-1 contains technical measures for the design of machines that transmit whole-body vibrations to humans.
Output date	DIN 76-1 (2004-06)	Date of publication which is made public in the DIN publication guide; this is the date at which time the standard becomes valid. DIN 76-1, which sets undercuts for metric ISO threads has been valid since June 2004 for example.

Types of standards and regulations (selection)

Type	Abbreviation	Explanation	Purpose and contents
International standards (ISO standards)	ISO	**I**nternational **O**rganization for **S**tandardization, Geneva (O and S are reversed in the abbreviation)	Simplifies the international exchange of goods and services as well as cooperation in scientific, technical and economic areas.
European standards (EN standards)	EN	**E**uropean Committee for Standardization (Comité Européen de **N**ormalisation), Brussels	Technical harmonization and the associated reduction of trade barriers for the advancement of the European market and the coalescence of Europe.
German standards (DIN standards)	DIN	**D**eutsches **I**nstitut für **N**ormung e.V., Berlin (German Institute for Standardization)	National standardization facilitates rationalization, quality assurance, environmental protection and common understanding in economics, technology, science, management and public relations.
	DIN EN	European standard for which the German version has attained the status of a German standard	
	DIN ISO	German standard for which an international standard has been adopted without change.	
	DIN EN ISO	European standard for which an international standard has been adopted unchanged and the German version has the status of a German standard.	
	DIN VDE	Printed publication of the VDE, which has the status of a German standard.	
VDI guidelines	VDI	**V**erein **D**eutscher **I**ngenieure e.V., Düsseldorf (Association of German Engineers)	These guidelines give an account of the current state of the art in specific subject areas and contain, for example, concrete procedural guidelines for the performing calculations or designing processes in mechanical or electrical engineering.
VDE printed-publications	VDE	**V**erband **d**er **E**lektrotechnik (German Association of Electrical Engineering) Elektronik Informationstechnik e.V., Frankfurt am Main (Association for Electronics Information Technology)	
DGQ publications	DGQ	**D**eutsche **G**esellschaft für **Q**ualität e.V., Frankfurt (German Society for Quality)	Recommendations in the area of quality technology.
REFA sheets	REFA	Verband für Arbeitsstudien (Association for Work Design) REFA e.V., Darmstadt (Industrial Organization and Corporate Development)	Recommendations in the area of production and work planning.

1 Technical Mathematics



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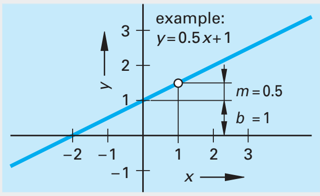

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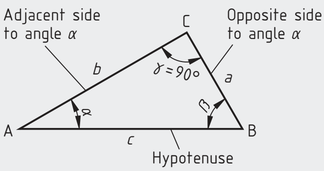

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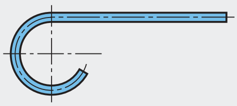

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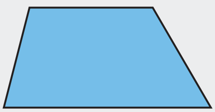

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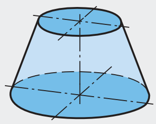

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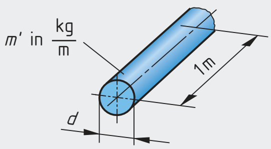

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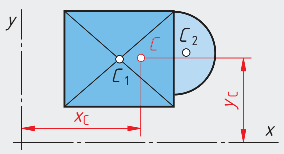

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Units of measurement

TM

SI[1] base quantities and base units

cf. DIN 1301-1 (2010-10), -2 (1978-02), -3 (1979-10)

Base quantity	Length	Mass	Time	Electric current	Thermo-dynamic temperature	Amount of substance	Luminous intensity
Base units	metre	kilogramme	second	ampere	kelvin	mole	candela
Unit symbol	m	kg	s	A	K	mol	cd

[1] The units for measurement are defined in the International System of Units SI (**S**ystème **I**nternational d'Unités). It is based on the seven basic units (SI units), from which other units are derived.

Base quantities, derived quantities and their units

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Length, Area, Volume, Angle					
Length	l	**metre**	m	1 m = 10 dm = 100 cm = 1000 mm 1 mm = 1000 µm 1 km = 1000 m	1 inch = 25.4 mm In aviation and nautical applications the following applies: 1 international nautical mile = 1852 m
Area	A, S	square metre are hectare	m² a ha	1 m² = 10 000 cm² = 1 000 000 mm² 1 a = 100 m² 1 ha = 100 a = 10 000 m² 100 ha = 1 km²	Symbol S only for cross-sectional areas Ar and hectare only for land
Volume	V	cubic metre litre	m³ l, L	1 m³ = 1000 dm³ = 1 000 000 cm³ 1 l = 1 L = 1 dm³ = 10 dl = 0.001 m³ 1 ml = 1 cm³	Mostly for fluids and gases
Plane angle (angle)	α, β, γ ...	radian degrees minutes seconds	rad ° ′ ″	1 rad = 1 m/m = 57.2957...° = 180°/π 1° = $\frac{\pi}{180}$ rad = 60′ 1′ = 1°/60 = 60″ 1″ = 1′/60 = 1°/3600	1 rad is the angle formed by the intersection of a circle around the centre of 1 m radius with an arc of 1 m length. In technical calculations instead of α = 33° 17′ 27.6″ it is better to use α = 33.291°.
Solid angle	Ω	steradian	sr	1 sr = 1 m²/m²	The solid angle of 1 sr encompasses a sphere of r = 1 m on the surface, which corresponds to the area of a spherical segment of A_O = 1 m².
Mechanics					
Mass	m	**kilogramme** gram megagram metric ton	kg g Mg t	1 kg = 1000 g 1 g = 1000 mg 1 t = 1000 kg = 1 Mg 0.2 g = 1 ct	In everyday language, the mass of a solid is also referred to as weight. Mass for precious stones in carat (ct).
Linear mass density	m'	kilogramme per metre	kg/m	1 kg/m = 1 g/mm	For calculating the mass of bars, pro-files, pipes.
Area mass density	m''	kilogramme per square metre	kg/m²	1 kg/m² = 0.1 g/cm²	To calculate the mass of sheet metal.
Density	ϱ	kilogramme per cubic metre	kg/m³	1000 kg/m³ = 1 metric t/m³ = 1 kg/dm³ = 1 g/cm³ = 1 g/ml = 1 mg/mm³	Density = Mass of a substance per volume unit For homogenous solids, the density is a location-independent quantity.

Units of measurement

Quantities and units (continued)

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Mechanics					
Moment of inertia, 2nd moment of mass	J	kilogramme x square metre	kg · m²	The following applies to homogenous full cylinders with a mass m and radius r. $J = \frac{1}{2} \cdot m \cdot r^2$	The moment of inertia indicates the resistance of a rigid homogenous solid against the change in its rotational movement along the axis of rotation.
Force Weight	F F_W, W	newton	N	$1\ \text{N} = 1\ \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 1\ \frac{\text{J}}{\text{m}}$ 1 MN = 10³ kN = 1 000 000 N	The force 1 N effects a change in velocity of 1 m/s in 1 s in a 1 kg mass.
Torque Bend. mom. Tors. mom.	M M_b M_T, T	newton x metre	N · m	$1\ \text{N} \cdot \text{m} = 1\ \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$	1 N · m is the moment that a force of 1 N effects with a lever arm of 1 m.
Momentum	p	kilogramme x metre per second	kg· m/s	1 kg · m/s = 1 N · s	The momentum is the product of the mass times velocity. It has the direction of the velocity.
Pressure Mechanical stress	p σ, τ	pascal newton per square millimetre	Pa N/mm²	1 Pa = 1 N/m² = 0.01 mbar 1 bar = 100 000 N/m² = 10 N/cm² = 10⁵ Pa 1 mbar = 1 hPa 1 N/mm² = 10 bar = 1 MN/m² = 1 MPa 1 daN/cm² = 0.1 N/mm²	Pressure refers to the force per unit area. For gage pressure, the symbol p_g is used (DIN 1314). 1 bar = 14.5 psi (pounds per square inch
Second moment of area	I	metre to the fourth power centimetre to the fourth power	m⁴ cm⁴	1 m⁴ = 100 000 000 cm⁴	Previously: Geometrical moment of inertia
Energy, Work, Quantity of heat	E, W	joule	J	1 J = 1 N · m = 1 W · s = 1 kg · m²/s²	Joule for all forms of energy, kW· h preferred for electrical energy.
Power, Heat flow	P Φ	watt	W	1 W = 1 J/s = 1 N · m/s = 1 V · A = 1 m² · kg/s³	Power describes the work which is achieved within a specific time.
Time					
Time, Time span, Duration	t	**seconds** minutes hours day year	s min h d a	1 min = 60 s 1 h = 60 min = 3600 s 1 d = 24 h = 86 400 s	3 h means a time span (3 hrs.), 3^h means a point in time (3 o'clock). If points in time are written in mixed form, e.g. $3^h 24^m 10^s$, the symbol min can be shortened to m.
Frequency	f, ν	hertz	Hz	1 Hz = 1/s	1 Hz ≙ 1 cycle in 1 second.
Rotational speed, Rotational frequency	n	1 per second 1 per minute	1/s 1/min	1/s = 60/min = 60 min⁻¹ $1/\text{min} = 1\ \text{min}^{-1} = \frac{1}{60\ \text{s}}$	The number of revolutions per unit of time gives the revolution frequency, also called rpm.
Velocity	v	metre per second metre per minute kilometre per hour	m/s m/min km/h	1 m/s = 60 m/min = 3.6 km/h $1\ \text{m/min} = \frac{1\ \text{m}}{60\ \text{s}}$ $1\ \text{km/h} = \frac{1\ \text{m}}{3.6\ \text{s}}$	Speed for nautical journeys in knots (kn): 1 kn = 1.852 km/h mile per hour = 1 mile/h = 1 mph 1 mph = 1.60934 km/h
Angular velocity	ω	1 per second radians per second	1/s rad/s	$\omega = 2\ \pi \cdot n$	For a rpm of n = 2/s the angular velocity $\omega = 4\ \pi$/s.
Acceleration	a, g	metre per second squared	m/s²	$1\ \text{m/s}^2 = \frac{1\ \text{m/s}}{1\ \text{s}}$	Symbol g only for acceleration due to gravity. $g = 9.81\ \text{m/s}^2 \approx 10\ \text{m/s}^2$

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Units of measurement

Quantities and units (continued)

Quantity	Symbol	Unit Name	Unit Symbol	Relationship	Remarks Examples of application
Electricity and magnetism					
Electric current Electro-motive force Elect. resistance Elect. conductance	I U R G	**ampere** volt ohm siemens	A V Ω S	 1 V = 1 W/1 A = 1 J/C 1 Ω = 1 V/1 A 1 S = 1 A/1 V = 1/Ω	A moving electrical charge is called electricity. The electromotive force is equal to the potential difference between two points in an electric field. The reciprocal of the electrical resistance is called the electrical conductivity.
Specific resistance Conductivity	ϱ $\gamma, \varkappa$	ohm x metre siemens per metre	Ω · m S/m	10^{-6} Ω · m = 1 Ω · mm^2/m	$\varrho = \frac{1}{\varkappa}$ in $\frac{\Omega \cdot mm^2}{m}$ $\varkappa = \frac{1}{\varrho}$ in $\frac{m}{\Omega \cdot mm^2}$
Frequency	f	hertz	Hz	1 Hz = 1/s 1000 Hz = 1 kHz	Frequency of public electric utility: EU 50 Hz, USA/Canada 60 Hz
Elect. Work	W	joule	J	1 J = 1 W · s = 1 N · m 1 kW · h = 3.6 MJ 1 W · h = 3.6 kJ	In atomic and nuclear physics the unit eV (electron volt) is used.
Phase difference	φ	–	–	for alternating current: $\cos\varphi = \frac{P}{U \cdot I}$	The angle between current and voltage in an inductive or capacitive load.
Elect. field strength Elect. charge Elect. capacity Inductance	E Q C L	volts per metre coulomb farad henry	V/m C F H	 1 C = 1 A · 1 s; 1 A · h = 3.6 kC 1 F = 1 C/V 1 H = 1 V · s/A	$E = \frac{F}{Q}$, $C = \frac{Q}{U}$, $Q = I \cdot t$
Power Effective power	P	watt	W	1 W = 1 J/s = 1 N · m/s = 1 V · A	In electrical power engineering: Apparent power S in V · A
Thermodynamics and heat transfer					
Quantity	**Symbol**	**Unit Name**	**Unit Symbol**	**Relationship**	**Remarks Examples of application**
Thermo-dynamic temperature Celsius temperature	T, Θ t, ϑ	**kelvin** degrees Celsius	K °C	0 K = – 273.15 °C 0 °C = 273.15 K 0 °C = 32 °F 0 °F = – 17.77 °C	Kelvin (K) and degrees Celsius (°C) are used for temperatures and temperature differences. $t = T - T_0$; T_0 = 273.15 K Conversion in °F: page 51
Quantity of heat	Q	joule	J	1 J = 1 W · s = 1 N · m 1 kW · h = 3600000 J = 3,6 MJ	1 kcal ≙ 4.1868 kJ
Net calorific value	H_u	joule per kilogramme joule per cubic metre	J/kg J/m^3	1 MJ/kg = 1000000 J/kg 1 MJ/m^3 = 1000000 J/m^3	Thermal energy released per kg fuel (or for each m^3) minus the heat of vaporisation of the water vapour contained in the exhaust gases.

Non-SI units

Length	Area	Volume	Mass	Energy, power
1 inch (in) = 25.4 mm	1 sq.in = 6.452 cm^2	1 cu.in = 16.39 cm^3	1 oz = 28.35 g	1 PSh = 0.735 kWh
1 foot (ft) = 0.3048 m	1 sq.ft = 9.29 dm^2	1 cu.ft = 28.32 dm^3	1 lb = 453.6 g	1 PS = 0.7355 kW
1 yard (yd) = 0.9144 m	1 sq.yd = 0.8361 m^2	1 cu.yd = 764.6 dm^3	1 t = 1000 kg	1 kcal = 4186.8 Ws
1 nautical mile = 1.852 km	1 acre = 4046.873 m^2	1 gallon (US) = 3.785 l	1 short ton = 907.2 kg	1 kcal = 1.166 Wh
1 land mile = 1.6093 km	**Pressure, force**	1 gallon (UK) = 4.546 l	1 Karat = 0.2 g	1 kpm/s = 9.807 W
	1 bar = 14.5 pound/in^2	1 barrel (US) = 158.9 l	1 pound/in^3 = 27.68 g/cm^3	1 Btu = 1055 Ws
	1 N/mm^2 = 145.038 pound/in^2	1 barrel (UK) = 159.1 l		1 bhp = 745.7 W

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Formula symbols, mathematical symbols

Formula symbols

cf. DIN 1304-1 (1994-03)

Formula symbol	Meaning	Formula symbol	Meaning	Formula symbol	Meaning
Length, Area, Volume, Angle					
l	Length	r, R	Radius	α, β, γ	Planar angle
w	Width	d, D	Diameter	Ω	Solid angle
h	Height	A, S	Area, Cross-sectional area	λ	Wave length
s	Linear distance	V	Volume		
Mechanics					
m	Mass	F	Force	G	Shear modulus
m'	Linear mass density	F_W, W	Gravitational force, Weight	μ, f	Coefficient of friction
m''	Area mass density	T	Torque	W	Section modulus
ϱ	Density	M_T, T	Torsional moment	I	2nd moment of mass
J	Moment of inertia	M_b	Bending moment	W, E	Work, Energy
p	Pressure	σ	Normal stress	W_p, E_p	Potential energy
p_{abs}	Absolute pressure	τ	Shear stress	W_k, E_k	Kinetic energy
p_{amb}	Ambient pressure	ε	Strain	P	Power
p_g	Gage pressure	E	Modulus of elasticity	η	Efficiency
Time					
t	Time, Duration	f, ν	Frequency	a	Acceleration
T	Cycle duration	v, u	Velocity	g	Local gravitational acceleration
n	Revolution frequency, Speed	ω	Angular velocity	α	Angular acceleration
				$Q, \dot{V}, q_v$	Volumetric flow rate
Electricity					
Q	Electric charge, Quantity of electricity	L	Inductance	X	Reactance
E	Electromotive force	R	Resistance	Z	Impedance
C	Capacitance	ϱ	Specific resistance	φ	Phase difference
I	Electric current	$\gamma, \varkappa$	Electrical conductivity	N	Number of turns
Heat					
T, Θ	Thermodynamic temperature	Q	Heat, Quantity of heat	$\Phi, \dot{Q}$	Heat flow
$\Delta T, \Delta t, \Delta\vartheta$	Temperature difference	λ	Thermal conductivity	a	Thermal diffusivity
t, ϑ	Celsius temperature	α	Heat transition coefficient	c	Specific heat capacity
α_l, α	Coefficient of linear expansion	k	Heat transmission coefficient	H_{net}	Net calorific value
Light, Electromagnetic radiation					
E_v	Illuminance	f	Focal length	I_e	Luminous intensity
		n	Refractive index	Q_e, W	Radiant energy
Acoustics					
p	Acoustic pressure	L_p	Acoustic pressure level	N	Loudness
c	Acoustic velocity	I	Sound intensity	L_N	Loudness level

Mathematical symbols

cf. DIN 1302 (1999-12)

Math. symbol	Spoken	Math. symbol	Spoken	Math. symbol	Spoken
$\approx$	approximately the same	$\sim$	proportional	log	logarithm (general)
$\triangleq$	corresponds to	a^x	a high x, xth of power of a	lg	common logarithm
…	and so forth	$\sqrt{\ }$	square root of	ln	natural logarithm
∞	infinity	$\sqrt[n]{\ }$	nth root of	e	Euler number (e = 2.718281…)
$=$	equal to	$\lvert x \rvert$	amount of x	sin	sine
$\neq$	not equal to	$\perp$	vertical to	cos	cosine
$\stackrel{def}{=}$	is the same by definition	$\parallel$	is parallel to	tan	tangent
$<$	less than	$\uparrow\uparrow$	parallel in same direction	cot	cotangent
$\leq$	less than or equal to	$\uparrow\downarrow$	parallel in opposite direction	(), [], { }	parentheses, brackets open and closed
$>$	greater than	$\sphericalangle$	angle		
$\geq$	greater than or equal to	$\triangle$	triangle	π	pi (circle constant = 3.14159 …)
$+$	plus	$\cong$	congruent to		
$-$	minus	Δx	delta x (difference between two values)	$\overline{AB}$	line segment AB
$\cdot$	times, multiplied by			$\overset{\frown}{AB}$	arc AB
–, /, :, ÷	over, divided by, per, to	%	percent, of a hundred	a′, a″	a prime, a double prime
Σ	sigma (summation)	‰	per mil, of a thousand	a_1, a_2	a sub 1, a sub 2

Formulas, equations, graphs

Formulas

TM

In most cases, the calculation of physical quantities is done with the help of formulas. They consist of:

- Formula symbols, e.g. v_c for cutting velocity, d for diameter, n for speed
- Operators (calculation rules), e.g. · for multiplication, + for addition, – for subtraction and — (fraction line) for division
- Constants, e.g. π (pi) = 3.14159 …
- Numbers, e.g. 10, 15 …

The formula symbols (page 13) are wildcards for quantities. When solving mathematical problems, the known quantities with their units are filled in the formulas. Before or during the calculation process, the units are converted in a way that

- the calculation becomes feasible or
- the result comprises the required unit.

Most quantities and units are standardised (page 10).

The **result** is always a **numerical value** accompanied by a **unit,** e.g. 4.5 m, 15 s

Formula for cutting velocity

$$v_c = \pi \cdot d \cdot n$$

Example:

What is the cutting velocity v_c in m/min for d = 200 mm and n = 630/min?

$$v_c = \pi \cdot d \cdot n = \pi \cdot 200\ \text{mm} \cdot 630\ \frac{1}{\text{min}} = \pi \cdot 200\ \text{mm} \cdot \frac{1\ \text{m}}{1000\ \text{mm}} \cdot 630\ \frac{1}{\text{min}} = \mathbf{395.84\ \frac{m}{min}}$$

Numerical value equations

Numerical value equations or numerical equations are formulas in which the typical conversions of units have already been integrated. The following should be noted when using equations:

The numerical values of the individual quantities may only be used in combination with the designated unit.

- The units are not carried along in the calculation.
- The unit of the quantity to be obtained is predetermined.

Example:

What is the torque T of an electrical motor with a driving power of P = 15 kW and a speed of n = 750/min?

$$T = \frac{9550 \cdot P}{n} = \frac{9550 \cdot 15}{750}\ \text{N} \cdot \text{m} = \mathbf{191\ N \cdot m}$$

Numerical value equation for torque

$$T = \frac{9550 \cdot P}{n}$$

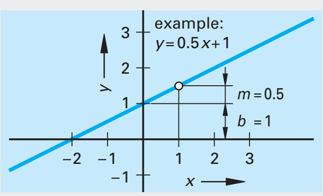
Designated unit		
Designation		Unit
T	Torque	N · m
P	Power	kW
n	Speed	1/min

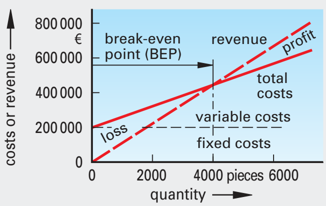
Equations and graphs

In functional equations, y is the function of x, with x as an independent and y as a dependent variable. The number pairs (x, y) of a value table form a graph in the x-y system of coordinates.

Assigned function

$$y = f(x)$$





1st example:

$y = 0.5\ x + 1$

x	–2	0	2	3
y	0	1	2	2.5

Linear function

$$y = m \cdot x + b$$

2nd example:

Cost function and revenue function

C_t = 60 €/piece · Q + 200 000 €
R = 110 €/piece · Q

Q	0	4 000	6 000
C_t	200 000	440 000	560 000
R	0	440 000	660 000

C_t total costs → dependent variable
Q quantity → independent variable
C_f fixed costs → y coordinate section
C_v variable costs → gradient of the function
R revenue → dependent variable

Examples:

Cost function

$$C_t = C_V \cdot Q + C_f$$

Revenue function

$$R = R/\text{piece} \cdot Q$$

Transformation of formulas

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Transformation of formulas

Formulas and numerical equations are transformed so that the quantity to be obtained stands alone on the left side of the equation. The value of the left side and right side of the formula must not change during the transformation. The following rule applies to all steps of the formula transformation:

Changes applied to the left formula side	=	Changes applied to the right formula side

Formula

$$P = \frac{F \cdot s}{t}$$

left side of the formula	=	right side of the formula

To be able to trace each step of the transformation, it is useful to mark it to the right next to the formula:

$| \cdot t$ → both sides of the formula are multiplied by t.

$| : F$ → both sides of the formula are divided by F.

Transformations of sums

Example: Formula $L = l_1 + l_2$, transformation to find l_2

1	$L = l_1 + l_2$	$\| - l_1$	subtract l_1	3	$L - l_1 = l_2$		invert both sides
2	$L - l_1 = l_1 + l_2 - l_1$		perform subtraction	4	$l_2 = L - l_1$		transformed formula

Transformations of products

Example: Formula $A = l \cdot b$, transformation to find l

1	$A = l \cdot b$	$\| : b$	divide by b	3	$\frac{A}{b} = l$		invert both sides
2	$\frac{A}{b} = \frac{l \cdot b}{b}$		cancel b	4	$l = \frac{A}{b}$		transformed formula

Transformations of fractions

Example: Formula $n = \frac{l}{l_1 + s}$, transformation to find s

1	$n = \frac{l}{l_1 + s}$	$\| \cdot (l_1 + s)$	multiply with $(l_1 + s)$	4	$n \cdot l_1 - n \cdot l_1 + n \cdot s = l - n \cdot l_1$	$\| : n$	subtract divide by n
2	$n \cdot (l_1 + s) = \frac{l \cdot (l_1 + s)}{(l_1 + s)}$		reduce right-side of formula solve the term in brackets	5	$\frac{s \cdot n}{n} = \frac{l - n \cdot l_1}{n}$		cancel n
3	$n \cdot l_1 + n \cdot s = l$	$\| - n \cdot l_1$	subtract $- n \cdot l_1$	6	$s = \frac{l - n \cdot l_1}{n}$		transformed formula

Transformations of roots

Example: Formula $c = \sqrt{a^2 + b^2}$, transformation to find a

1	$c = \sqrt{a^2 + b^2}$	$\| ()^2$	square the formula	4	$a^2 = c^2 - b^2$	$\| \sqrt{\ }$	square equation
2	$c^2 = a^2 + b^2$	$\| - b^2$	subtract b^2	5	$\sqrt{a^2} = \sqrt{c^2 - b^2}$		simplify the expression
3	$c^2 - b^2 = a^2 + b^2 - b^2$		subtract, invert both sides	6	$a = \sqrt{c^2 - b^2}$		transformed formula

Quantities and units

TM

Numerical values and units

Physical quantity

10 mm

Numerical Unit value

Physical quantities, e.g. 125 mm, consist of a

- **Numerical value,** which is determined by measurement or calculation, and a
- **Unit,** e.g. m, kg

Units are standardised in accordance with DIN 1301-1 (page 10).

Very large or very small numerical values can be represented in a simplified way as decimal multiples or factors with the help of prefixes, e.g. 0.004 mm = 4 µm.

Decimal multiples or factors of units

cf. DIN 1301-2 (1978-02)

Prefix Symbol	Prefix Name	Power of ten	Mathematical designation	Examples
T	tera	10^{12}	trillion	12 000 000 000 000 N = 12 · 10^{12} N = 12 TN (teranewtons)
G	giga	10^{9}	billion	45 000 000 000 W = 45 · 10^{9} W = 45 GW (gigawatts)
M	mega	10^{6}	million	8 500 000 V = 8.5 · 10^{6} V = 8.5 MV (megavolts)
k	kilo	10^{3}	thousand	12 600 W = 12.6 · 10^{3} W = 12.6 kW (kilowatts)
h	hecto	10^{2}	hundred	500 l = 5 · 10^{2} l = 5 hl (hectolitres)
da	deca	10^{1}	ten	32 m = 3.2 ·10^{1} m = 3.2 dam (decametres)
–	–	10^{0}	one	1.5 m = 1.5 · 10^{0} m
d	deci	10^{-1}	tenth	0.5 l = 5 · 10^{-1} l = 5 dl (decilitres)
c	centi	10^{-2}	hundredth	0.25 m = 25 · 10^{-2} m = 25 cm (centimetres)
m	milli	10^{-3}	thousandth	0.375 A = 375 · 10^{-3} A = 375 mA (milliamperes)
µ	micro	10^{-6}	millionth	0.000 052 m = 52 · 10^{-6} m = 52 µm (micrometres)
n	nano	10^{-9}	billionth	0.000 000 075 m = 75 · 10^{-9} m = 75 nm (nanometres)
p	pico	10^{-12}	trillionth	0.000 000 000 006 F = 6 · 10^{-12} F = 6 pF (picofarads)

Conversion of units

Calculations with physical units are only possible if these units refer to the same base in this calculation. When solving mathematical problems, units often must be converted to basic units, e.g. mm in m, h in s, mm^2 in m^2. This is done with the help of conversion factors that represent the value 1 (coherent units).

Conversion factors for units (excerpt)

Quantity	Conversion factors, e. g.	Quantity	Conversion factors, e. g.
Length	$1 = \frac{10\ \text{mm}}{1\ \text{cm}} = \frac{1000\ \text{mm}}{1\ \text{m}} = \frac{1\ \text{m}}{1000\ \text{mm}} = \frac{1\ \text{km}}{1000\ \text{m}}$	Time	$1 = \frac{60\ \text{min}}{1\ \text{h}} = \frac{3600\ \text{s}}{1\ \text{h}} = \frac{60\ \text{s}}{1\ \text{min}} = \frac{1\ \text{min}}{60\ \text{s}}$
Areas	$1 = \frac{100\ \text{mm}^2}{1\ \text{cm}^2} = \frac{100\ \text{cm}^2}{1\ \text{dm}^2} = \frac{1\ \text{cm}^2}{100\ \text{mm}^2} = \frac{1\ \text{dm}^2}{100\ \text{cm}^2}$	Angle	$1 = \frac{60'}{1°} = \frac{60''}{1'} = \frac{3600''}{1°} = \frac{1°}{60\ \text{s}}$
Volume	$1 = \frac{1000\ \text{mm}^3}{1\ \text{cm}^3} = \frac{1000\ \text{cm}^3}{1\ \text{dm}^3} = \frac{1\ \text{cm}^3}{1000\ \text{mm}^3} = \frac{1\ \text{dm}^3}{1000\ \text{cm}^3}$	Inch	1 inch = 25.4 mm; 1 mm = $\frac{1}{25.4}$ inch

1st example:

Convert volume $V = 3416\ \text{mm}^3$ to cm^3.

Volume V is multiplied by a conversion factor. Its numerator has the unit cm^3 and its denominator the unit mm^3.

$$\boldsymbol{V} = 3416\ \text{mm}^3 = \frac{1\ \text{cm}^3 \cdot 3416\ \text{mm}^3}{1000\ \text{mm}^3} = \frac{3416\ \text{cm}^3}{1000} = \mathbf{3.416\ cm^3}$$

2nd example:

The angle size specification $\alpha = 42° 16'$ is to be expressed in degrees (°).

The partial angle 16′ must be converted to degrees (°). The value is multiplied by a conversion factor, the numerator of which has the unit degree (°) and the denominator the unit minute (′).

$$\alpha = 42° + 16' \cdot \frac{1°}{60'} = 42° + \frac{16 \cdot 1°}{60} = 42° + 0.267° = \mathbf{42.267°}$$

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Calculation with quantities, percentage and interest calculation

Calculation with quantities

Physical quantities are mathematically treated as products.

- **Adding and subtracting**

  Numerical values that have the same unit are added or subtracted and the unit is carried over to the result.

Example:

$L = l_1 + l_2 - l_3$ where $l_1 = 124$ mm, $l_2 = 18$ mm, $l_3 = 44$ mm; $L = ?$

$\mathbf{L} = 124\text{ mm} + 18\text{ mm} - 44\text{ mm} = (124 + 18 - 44)\text{ mm} = \mathbf{98\text{ mm}}$

- **Multiplying and dividing**

  The numerical values and the units correspond to the factors of products.

Example:

$F_1 \cdot l_1 = F_2 \cdot l_2$ where $F_1 = 180$ N, $l_1 = 75$ mm, $l_2 = 105$ mm; $F_2 = ?$

$$\mathbf{F_2} = \frac{F_1 \cdot l_1}{l_2} = \frac{180\text{ N} \cdot 75\text{ mm}}{105\text{ mm}} = 128.57\,\frac{\text{N} \cdot \text{mm}}{\text{mm}} = \mathbf{128.57\text{ N}}$$

- **Multiplying and dividing powers**

  Powers that have the same base are multiplied or divided by adding or subtracting their exponents.

Example:

$W = \frac{A \cdot a^2}{e}$ with $A = 15\text{ cm}^2$, $a = 7.5$ cm, $e = 2.4$ cm; $W = ?$

$$\mathbf{W} = \frac{15\text{ cm}^2 \cdot (7.5\text{ cm})^2}{2.4\text{ cm}} = \frac{15 \cdot 56.25\text{ cm}^{2+2}}{2.4\text{ cm}^1} = 351.56\text{ cm}^{4-1} = \mathbf{351.56\text{ cm}^3}$$

Rules for raising to powers

a base

$m, n \ldots$ exponents

Multiplying powers

$$a^2 \cdot a^3 = a^{2+3}$$

$$a^m \cdot a^n = a^{m+n}$$

Dividing powers

$$\frac{a^2}{a^3} = a^{2-3}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Special cases

$$a^{-2} = \frac{1}{a^2}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^1 = a \qquad a^0 = 1$$

Percentage calculation

The **percentage rate** indicates the part of the base value in hundredths.
The **base value** is the value from which the percentage is to be calculated.
The **percent value** is the amount representing the percentage of the base value.

P_r percentage rate, in percent $\quad$ P_v percent value $\quad$ B_v base value

Example:

Weight of raw part: 250 kg (base value); material loss of 2% (percentage rate); material loss in kg = ? (percent value)

$$\mathbf{P_v} = \frac{B_v \cdot P_r}{100\%} = \frac{250\text{ kg} \cdot 2\%}{100\%} = \mathbf{5\text{ kg}}$$

Percent value

$$P_v = \frac{B_v \cdot P_r}{100\%}$$

Interest calculation

P principle $\quad$ I interest $\quad$ t period in days, interest period

A amount accumulated $\quad$ r interest rate per year

1st example:

$P = 2800.00$ €; $r = 6\frac{\%}{\text{a}}$; $t = ½$ a; $I = ?$

$$I = \frac{2800.00\text{ €} \cdot 6\frac{\%}{\text{a}} \cdot 0.5\text{ a}}{100\%} = \mathbf{84.00\text{ €}}$$

2nd example:

$P = 4800.00$ €; $r = 5.1\frac{\%}{\text{a}}$; $t = 50$ d; $I = ?$

$$I = \frac{4800.00\text{ €} \cdot 5.1\frac{\%}{\text{a}} \cdot 50\text{ d}}{100\% \cdot 360\frac{\text{d}}{\text{a}}} = \mathbf{34.00\text{ €}}$$

Interest

$$I = \frac{P \cdot r \cdot t}{100\% \cdot 360}$$

1 interest year (1 a) = 360 days (360 d)

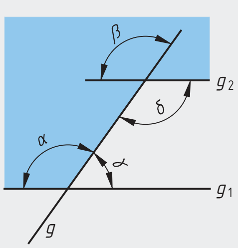
360 d = 12 months

1 interest month = 30 days

Types of angles, theorem of intersecting lines, angles in a triangle, pythagorean theorem

TM

Types of angles



g	straight line
g_1, g_2	parallel straight lines
α, β	corresponding angles
β, δ	opposite angles
α, δ	alternate angles
α, γ	adjacent angles

If two parallels are intersected by a straight line, there are geometrical interrelationships between the resulting angles.

Corresponding angles

$$\alpha = \beta$$

Opposite angles

$$\beta = \delta$$

Alternate angles

$$\alpha = \delta$$

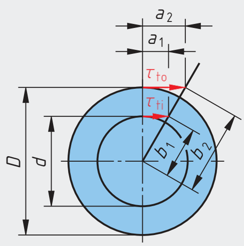
Adjacent angles

$$\alpha + \gamma = 180°$$

Theorem of intersecting lines

τ_{to} outer torsional stress

τ_{ti} inner torsional stress



If two intersecting lines are intercepted by a pair of parallels, the resulting segments form equal ratios.

$D = 40$ mm, $d = 30$ mm,

$\tau_{to} = 135$ N/mm²; $\tau_{ti} = ?$

$$\frac{\tau_{ti}}{\tau_{to}} = \frac{d}{D} \Rightarrow \tau_{ti} = \frac{\tau_{to} \cdot d}{D}$$

$$= \frac{135 \text{ N/mm}^2 \cdot 30 \text{ mm}}{40 \text{ mm}} = \mathbf{101.25\ N/mm^2}$$

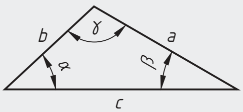
Theorem of intersecting lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{\frac{d}{2}}{\frac{D}{2}}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

$$\frac{b_1}{d} = \frac{b_2}{D}$$

Sum of angles in a triangle



a, b, c	sides of the triangle
α, β, γ	angles in the triangle

Example:

$\alpha = 21°$, $\beta = 95°$, $\gamma = ?$

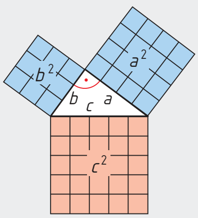
$\gamma = 180° - \alpha - \beta = 180° - 21° - 95° = \mathbf{64°}$

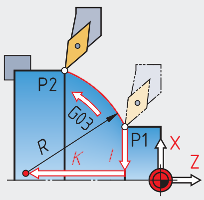
Sum of angles in a triangle

$$\alpha + \beta + \gamma = 180°$$

In every triangle, the sum of the interior angles equals 180°.

Pythagorean theorem





In a **right triangle** the square on the hypotenuse is equal to the sum of the squares on the sides meeting the right angle.

a	side
b	side
c	hypotenuse

1st example:

$c = 35$ mm; $a = 21$ mm; $b = ?$

$\mathbf{b} = \sqrt{c^2 - a^2} = \sqrt{(35 \text{ mm})^2 - (21 \text{ mm})^2} = \mathbf{28\ mm}$

2nd example:

CNC program where $R = 50$ mm and $I = 25$ mm.

$K = ?$

$c^2 = a^2 + b^2$

$R^2 = I^2 + K^2$

$K = \sqrt{R^2 - I^2} = \sqrt{50^2 \text{ mm}^2 - 25^2 \text{ mm}^2}$

$\mathbf{K} = \mathbf{43.3\ mm}$

Length of the hypotenuse

$$c^2 = a^2 + b^2$$

Square on the hypotenuse

$$c = \sqrt{a^2 + b^2}$$

Length of the sides

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

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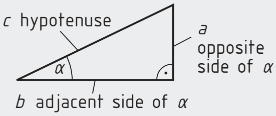
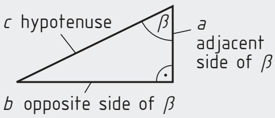
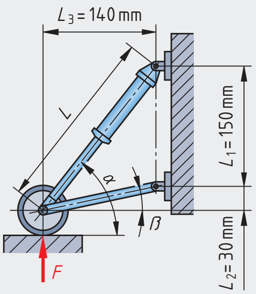
Functions of triangles

Functions of right triangles (trigonometric functions)







c	hypotenuse (longest side)
a, b	sides: – b is the adjacent side of α – a is the opposite side of α
α, β, γ	angles in the triangle, $\gamma = 90°$
sin	notation of sine
cos	notation of cosine
tan	notation of tangent
$\sin\alpha$	sine of angle α

1st example

L_1 = 150 mm, L_2 = 30 mm, L_3 = 140 mm; angle α = ?

$$\tan\alpha = \frac{L_1 + L_2}{L_3} = \frac{180\ \text{mm}}{140\ \text{mm}} = 1.286$$

Angle α = 52°

2nd example

L_1 = 150 mm, L_2 = 30 mm, α = 52°; Length of the shock absorber L = ?

$$L = \frac{L_1 + L_2}{\sin\alpha} = \frac{180\ \text{mm}}{\sin 52°} = \mathbf{228.42\ mm}$$

Trigonometric functions

sine	=	$\dfrac{\text{opposite side}}{\text{hypotenuse}}$
cosine	=	$\dfrac{\text{adjacent side}}{\text{hypotenuse}}$
tangent	=	$\dfrac{\text{opposite side}}{\text{adjacent side}}$
cotangent	=	$\dfrac{\text{adjacent side}}{\text{opposite side}}$

Relations applying to angle α:

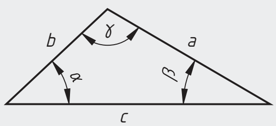
$\sin\alpha = \dfrac{a}{c}$	$\cos\alpha = \dfrac{b}{c}$	$\tan\alpha = \dfrac{a}{b}$

Relations applying to angle β:

$\sin\beta = \dfrac{b}{c}$	$\cos\beta = \dfrac{a}{c}$	$\tan\beta = \dfrac{b}{a}$

The calculation of an angle in degrees (°) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e. g. arc sine.

Functions of oblique triangles (law of sines, law of cosines)



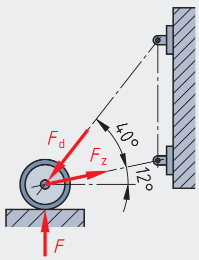
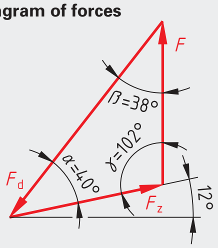


Diagram of forces



According to the law of sines, the ratios of the sides correspond to the sine of their opposite angles in the triangle. If one side and two angles are known, the other values can be calculated with the help of this function.

Side a → opposite angle α
Side b → opposite angle β
Side c → opposite angle γ

Example

F = 800 N, α = 40°, β = 38°; F_z = ?, F_d = ?

The forces are calculated with the help of the forces diagram.

$$\frac{F}{\sin\alpha} = \frac{F_z}{\sin\beta} \Rightarrow F_z = \frac{F \cdot \sin\beta}{\sin\alpha}$$

$$F_z = \frac{800\ \text{N} \cdot \sin 38°}{\sin 40°} = \mathbf{766.24\ N}$$

$$\frac{F}{\sin\alpha} = \frac{F_d}{\sin\varphi} \Rightarrow F_d = \frac{F \cdot \sin\varphi}{\sin\alpha}$$

$$F_d = \frac{800\ \text{N} \cdot \sin 102°}{\sin 40°} = \mathbf{1217.38\ N}$$

The calculation of an angle in degrees (°) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e.g. arc cos.

Law of sines

$$a : b : c = \sin\alpha : \sin\beta : \sin\gamma$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

There are many transformation options:

$$a = \frac{b \cdot \sin\alpha}{\sin\beta} = \frac{c \cdot \sin\alpha}{\sin\gamma}$$

$$b = \frac{a \cdot \sin\beta}{\sin\alpha} = \frac{c \cdot \sin\beta}{\sin\gamma}$$

$$c = \frac{a \cdot \sin\gamma}{\sin\alpha} = \frac{b \cdot \sin\gamma}{\sin\beta}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$$
$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$$
$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$$

Transformation, e. g.

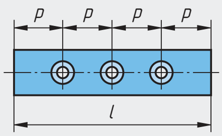
$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$$

Division of lengths, arc length, composite length

TM

Sub-dividing length

Edge distance = spacing



l total length
p division
n number of holes

Example:

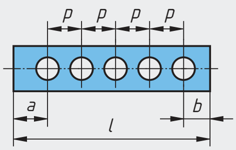
l = 2 m; n = 24 holes; p = ?

$\mathbf{p} = \frac{l}{n+1} = \frac{2000 \text{ mm}}{24+1} = \mathbf{80\ mm}$

Pitch

$$p = \frac{l}{n+1}$$

Edge distance ≠ spacing



l total length
p spacing
n number of holes
a, b edge distances

Example:

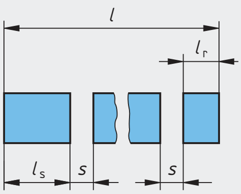
l = 1950 mm; a = 100 mm; b = 50 mm;
n = 25 holes; p = ?

$\mathbf{p} = \frac{l-(a+b)}{n-1} = \frac{1950 \text{ mm} - 150 \text{ mm}}{25-1} = \mathbf{75\ mm}$

Pitch

$$p = \frac{l-(a+b)}{n-1}$$

Subdividing into pieces



l bar length
z number of pieces
l_s piece length
s saw cutting width
l_r remaining length

Example:

l = 6 m; l_s = 230 mm; s = 1.2 mm; z = ?; l_r = ?

$\mathbf{z} = \frac{l}{l_s+s} = \frac{6000 \text{ mm}}{230 \text{ mm} + 1.2 \text{ mm}} = \mathbf{25.95 = 25\ pieces}$

$\mathbf{l_r} = l - z \cdot (l_s + s) = 6000 \text{ mm} - 25 \cdot (230 \text{ mm} + 1.2 \text{ mm})$
$= \mathbf{220\ mm}$

Number of pieces

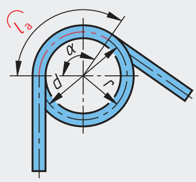
$$z = \frac{l}{l_s+s}$$

Remaining length

$$l_r = l - z \cdot (l_s + s)$$

Arc length

Example: Torsion spring



l_a arc length
r radius
α angle at centre
d diameter

Example:

r = 36 mm; α = 120°; l_a = ?

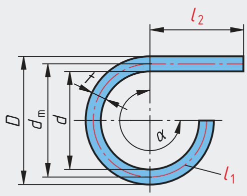
$\mathbf{l_a} = \frac{\pi \cdot r \cdot \alpha}{180°} = \frac{\pi \cdot 36 \text{ mm} \cdot 120°}{180°} = \mathbf{75.36\ mm}$

Arc length

$$l_a = \frac{\pi \cdot r \cdot \alpha}{180°}$$

$$l_a = \frac{\pi \cdot d \cdot \alpha}{360°}$$

Composite length



D outside diameter
d_m mean diameter
l_1, l_2 section lengths
α angle at centre
d inside diameter
t thickness
L composite length

Example (composite length, picture left):

D = 360 mm; t = 5 mm; α = 270°; l_2 = 70 mm;
d_m = ?; L = ?

$\mathbf{d_m} = D - t = 360 \text{ mm} - 5 \text{ mm} = \mathbf{355\ mm}$

$\mathbf{L} = l_1 + l_2 = \frac{\pi \cdot d_m \cdot \alpha}{360°} + l_2$

$= \frac{\pi \cdot 355 \text{ mm} \cdot 270°}{360°} + 70 \text{ mm} = \mathbf{906.45\ mm}$

Composite length

$$L = l_1 + l_2 + \ldots$$